Vectors Cheat Sheet

Vector Magnitude

$\|\langle a, b\rangle\| = \sqrt{a^2 + b^2}$

Unit Vector (Normalizing a Vector)

Divide by the magnitude: $\frac{\vec{u}}{\|\vec{u}\|}$

Adding Vectors

$\langle a, b\rangle + \langle c, d\rangle = \langle a + c, b + d\rangle$

$(a\hat{\imath} + b\hat{\jmath}) + (c\hat{\imath} + d\hat{\jmath}) = (a + c)\hat{\imath} + (b + d)\hat{\jmath}$

Scalar Multiplication

$a\langle b, c\rangle = \langle ab, ac\rangle$

$a(b\hat{\imath} + c\hat{\jmath}) = ab\hat{\imath} + ac\hat{\jmath}$

Dot Products

$\langle a, b\rangle \cdot \langle c, d\rangle = ac + bd$

$(a\hat{\imath} + b\hat{\jmath}) \times (c\hat{\imath} + d\hat{\jmath}) = ac + bd$

Alternative formula: $\vec{u} \cdot \vec{v} = \|\vec{u}\|\ \|\vec{v}\| \cos\theta$

Cross Products

$\langle a, b, c\rangle \times \langle d, e, f\rangle = \langle bf - ec,\quad -(af - dc),\quad ae - db\rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a & b & c \\ d & e & f \end{vmatrix} = (bf - ec)\hat{\imath} - (af - dc)\hat{\jmath} + (ae - db)\hat{k}$$

Magnitude of the cross product: $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\ \|\vec{v}\|\ \sin\theta$

Projections

$proj_u\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{u}$ $\qquad$ $proj_v\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v}$

Vector Angles

Finding the direction of a single vector: $\tan\theta = \frac{y}{x} = \frac{\hat{\jmath}\ term}{\hat{\imath}\ term}$

Finding the angle separating two vectors: $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\ \|\vec{v}\|}$