

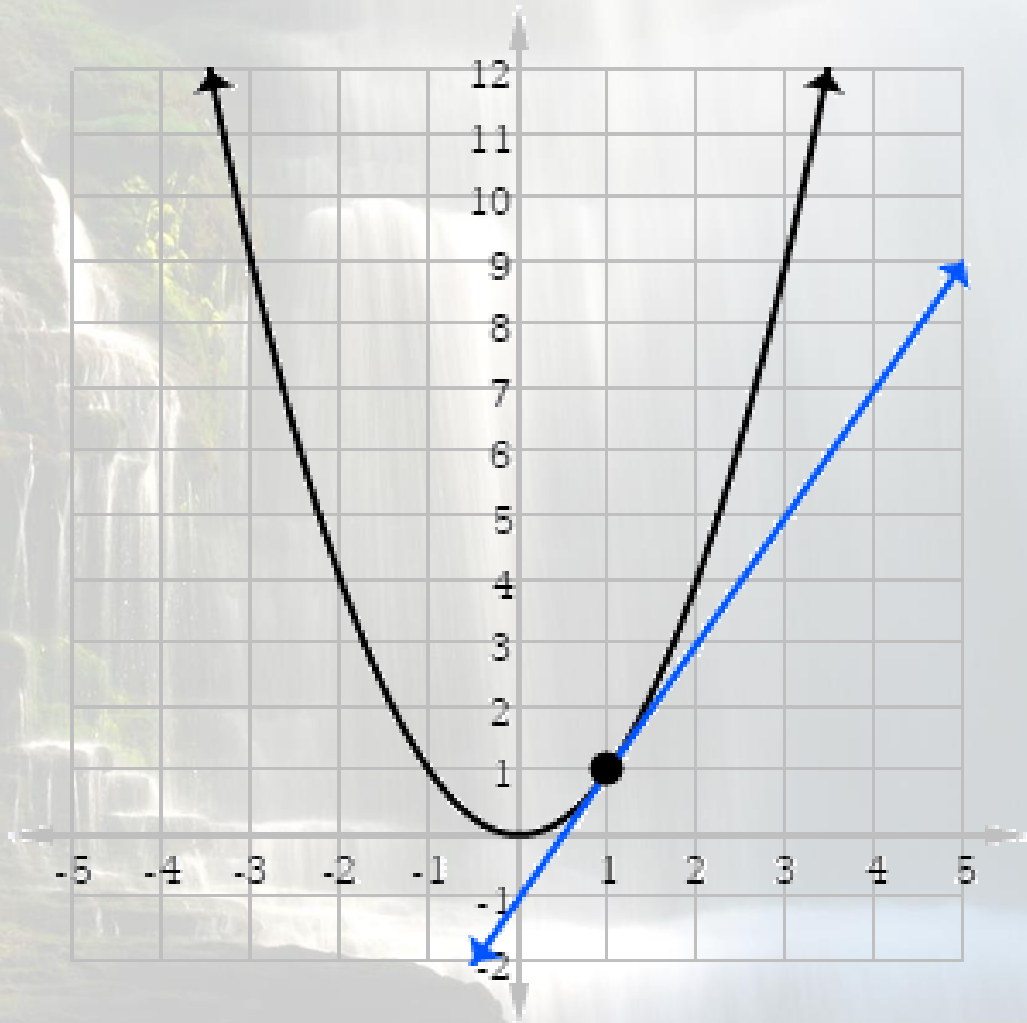


# *The* **Difference Quotient**

# The Difference Quotient

- Derive the difference quotient using the slope formula.
- Evaluate the difference quotient to find the derivative in general and at a specific point.
- Language Objectives:
  - Explain the relationship between a secant line and a tangent line.
  - Explain how a limit is involved in the formation of a tangent line.

# Introduction



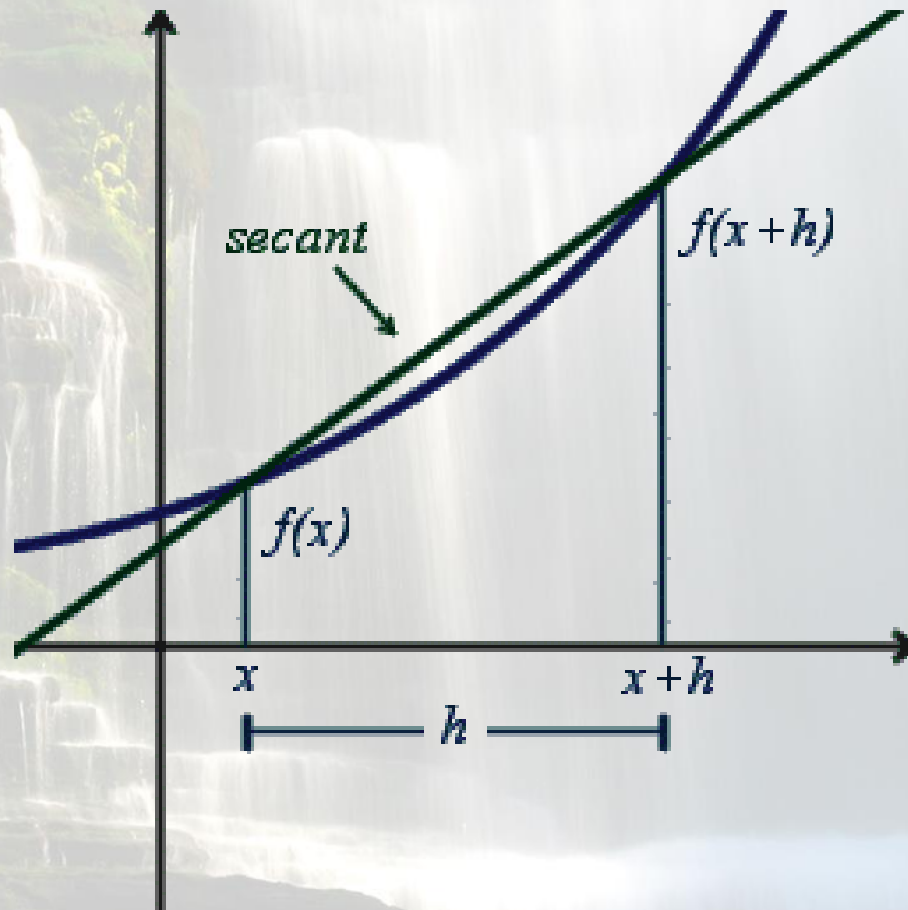
# Considering the Slope formula

- Slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

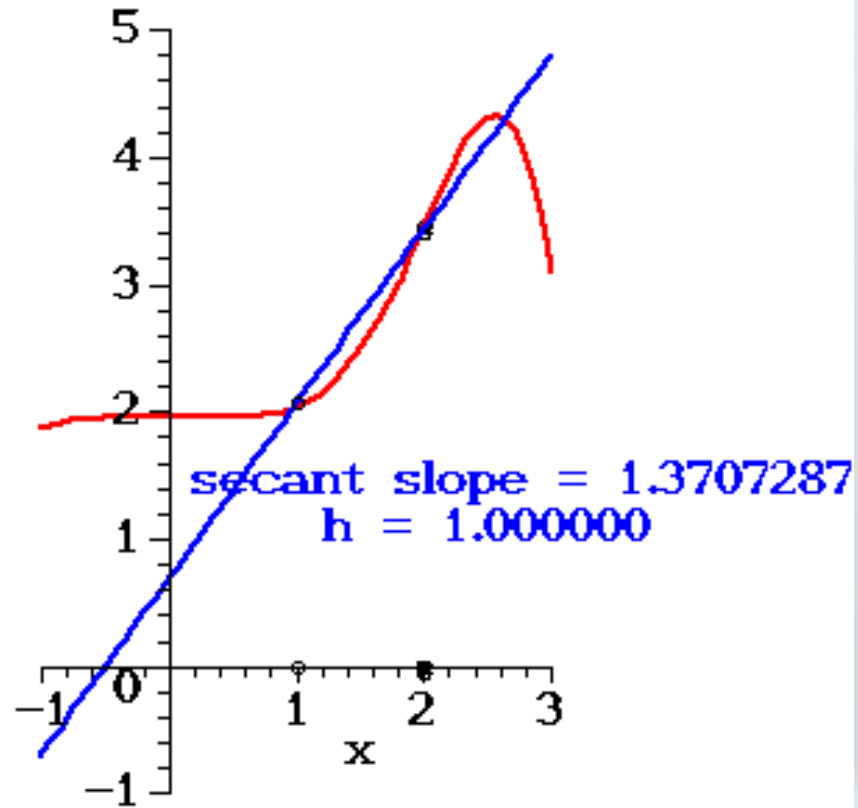
$$= \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{f(x + h) - f(x)}{h}$$

# What is $h$ ?



<http://mesacc.edu/~davvu41111/secanttangent.gif>

Secant to Tangent Animation  
 $f(x) = (1/10)x^4 \sin(x) + 2$



# We are approaching something

- *What are we dealing with?*

- *A limit!*

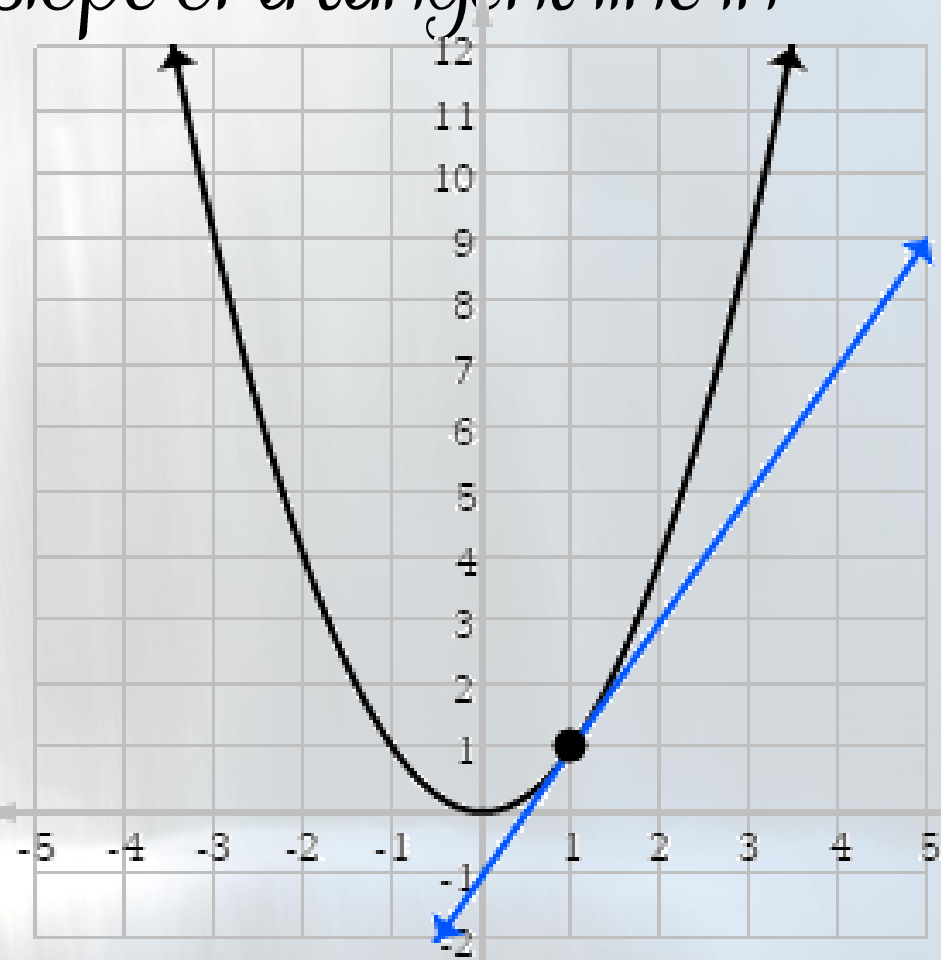
- *Difference quotient:*

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

# Write about it

- Explain the formula for slope of a tangent line in your own words.

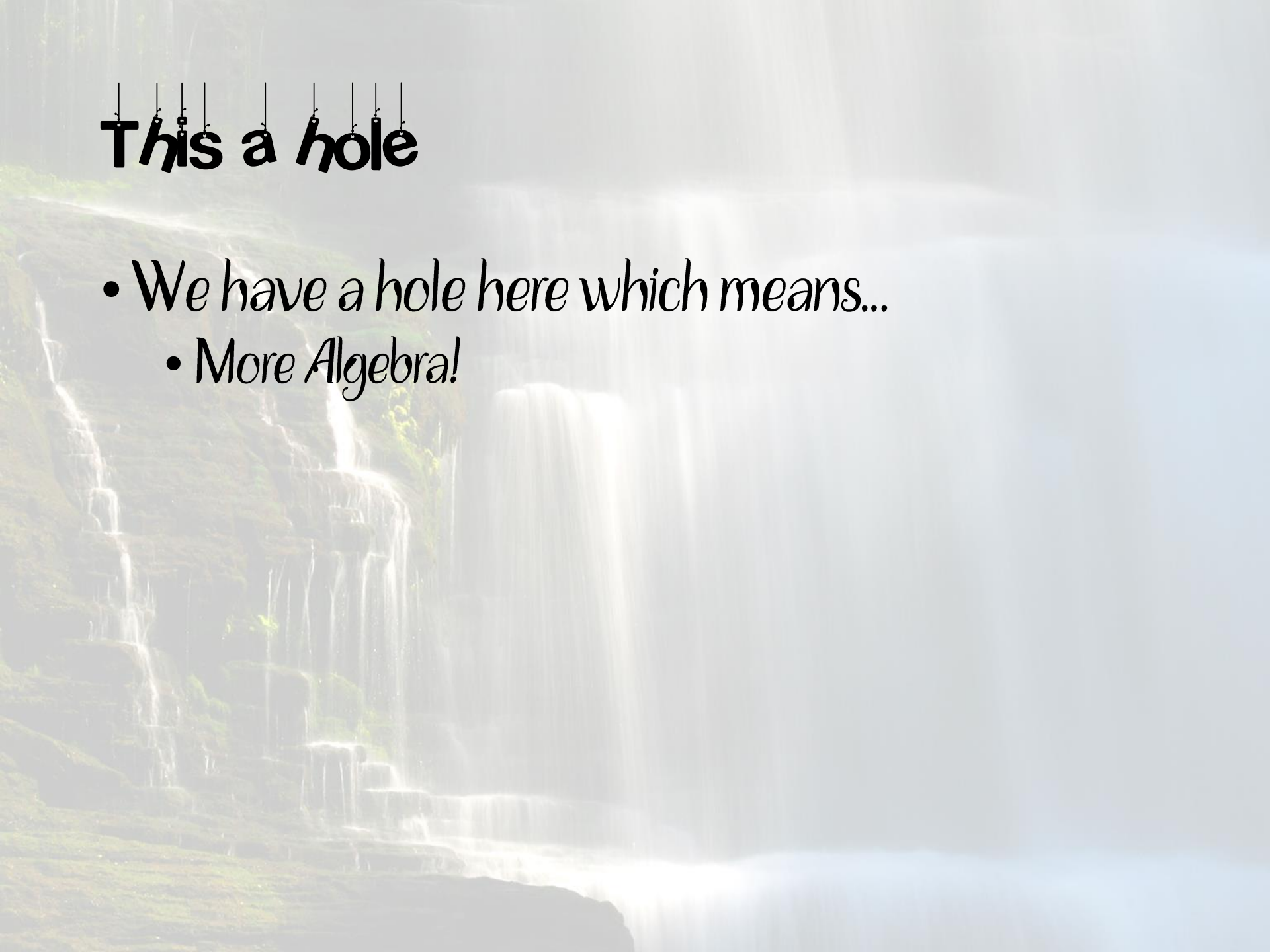
$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$





# Exploring this

- *An example... Use the difference quotient to find the slope of the tangent line for  $f(x) = 3x + 2$ .*
- *What do we always do first when working with a limit and an equation?*



*This a hole*

- *We have a hole here which means...*
  - *More Algebra!*

## Example 2

- Use the difference quotient to find the slope of the tangent line at  $x = 3$  for the function  $f(x) = x^2$ .

## Example 3

- Use the difference quotient to find  $f'(x)$  for the function  $f(x) = -x^2 + 7x$ .

## Example 4

- Find  $f'(x)$  if  $f(x) = 2x^2 - x$ .

# Example 5

- Find  $f'(3)$  if  $f(x) = 3x^2 + 5$

# Setting Up Difference Quotients

- *Set up, but don't simplify, the difference quotients for the following...*

- $f(x) = 2x^2 - 3x + 1$

- $f(x) = \sin x$

- $f(x) = \frac{1}{x^2}$

# Setting Up Difference Quotients

- *Set up, but don't simplify, the difference quotients for the following...*

- $f(x) = 4x^3 - 3x + 1$  at  $x = 0$

- $f(x) = \sin x$  at  $x = \frac{\pi}{2}$

- $f(x) = \frac{1}{x^2}$  at  $x = -1$