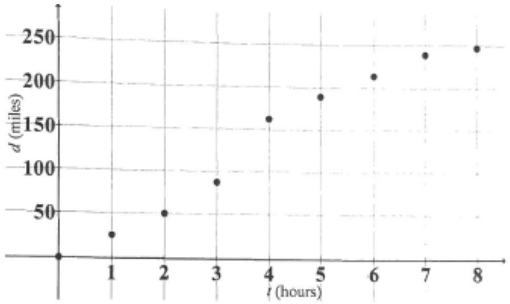
Slope Exploration

A family drives from their home in the city to a cabin in the woods. Before they begin, they reset the trip odometer in their car. Then, every hour, they record the trip odometer reading. The following graph shows the results.



1. Draw a line segment on the graph from the point at $t_0 = 0$ to the point $t_1 = 8$.

   a. What is the slope (m) of this line? Include units in your answer.

   b. Describe in words what this slope (m) tells you about the movement of the car throughout this time period.

2. Draw a line segment on the graph from the point $t_0 = 0$ to the point $t_1 = 2$.

   a. What is the slope of this line? Include units in your answer.

   b. Describe in words what this slope tells you about the movement of the car throughout this time period.

3. On the graph, draw a line segment between the points that mark the one hour during which the car was likely traveling on an interstate highway. Explain your reasoning.

4. In general, what information is conveyed by the slope of a line between any two points on a distance versus time function?

A car starts at rest at $t = 0$. Assume the distance the car travels is always a function of time according to the equation $d(t) = 3t^2$ with units: d (in meters) and t (in seconds) .

5. Sketch this graph.
6. Sketch a line segment connecting the graph from the point at $t = 1$ to the point at $t = 3$.

7. The slope of the straight line in your graph tells you the **<u>average</u>** velocity of the car between $t = 1$ and $t = 3$ seconds. Is this consistent with your answers to the previous questions? If not, discuss with your group and resolve any differences.

8. Circle the formula that gives the average velocity of the car between a time of interest which we will call a and a later time t.

$$v_{avg} = 3t^2 - 3a^2 \qquad v_{avg} = \frac{3t^2 - 3a^2}{t - a} \qquad v_{avg} = \frac{3t^2}{t}$$

9. Use the formula you chose to confirm that the average velocity of the car is 12 m/s if you set $a = 1$ and $t = 3$. Show your work.

10. Consider the points on the curve at $(1,3)$ and $(1.24, 3.2)$.

    a. Add a straight line to the graph that pass through these two points (note: the slope of this line is 6.6 m/s).

    b. Let's think of the instantaneous velocity of the car at time t as being the speedometer reading at time t. Which slope (12 m/s or 6.6 m/s) is a better estimate of the instantaneous velocity at our time of interest, $a = 1$ second? Explain your reasoning.

11. A student says that using $a = 1$ and $t = 1.1$ in the formula $v_{avg} = \frac{3t^2 - 3a^2}{t - a}$ generates an estimate of the instantaneous velocity of the car at $a = 1$ second that is even better than 6.6 m/s.

    a. Do you agree or disagree? Explain your reasoning.

    b. Choose a value of t that would give an even better estimate of the instantaneous velocity of the car at $a = 1$ second.

    c. Does your answer in part b give you the <u>very best</u> estimate? If not, describe in words how you can use the formula to generate increasingly better estimates for the instantaneous velocity of the car at $a = 1$ second.

12. What is the value of the function $v_{avg} = \frac{3t^2 - 3a^2}{t - a}$ when $a = t$? (e.g. if $a = 1$ and $t = 1$)