Continuity and the Intermediate Value Theorem

Continuity and the IVT

- Use the definition to determine whether a function is continuous at a given point and justify the reasoning.
- Use the Intermediate Value Theorem to justify claims.

- Language Objectives:
  - Define continuity both using the definition and visually.
  - Justify whether a function is continuous in written language.

What is Continuity?

- How do we know if something is continuous?

What is continuity?

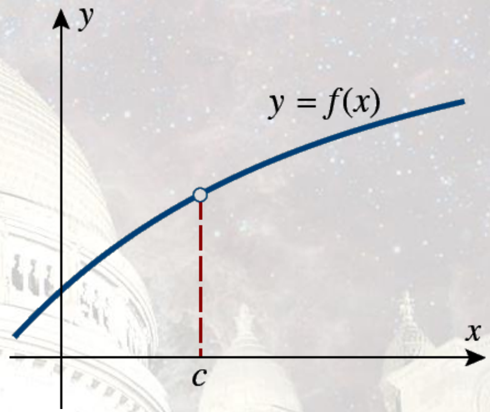
- Something that is unbroken.
- The function passes through an infinite number of points without skipping any of them.
- There are no wormholes.

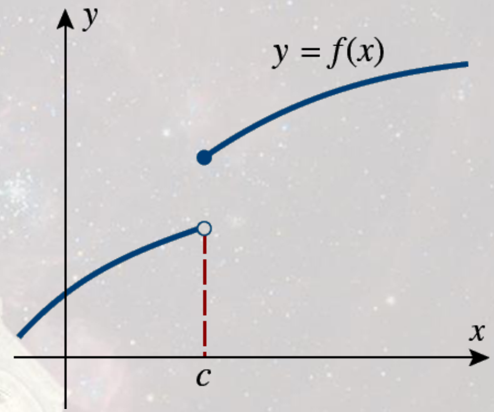
The Mathematical Definition

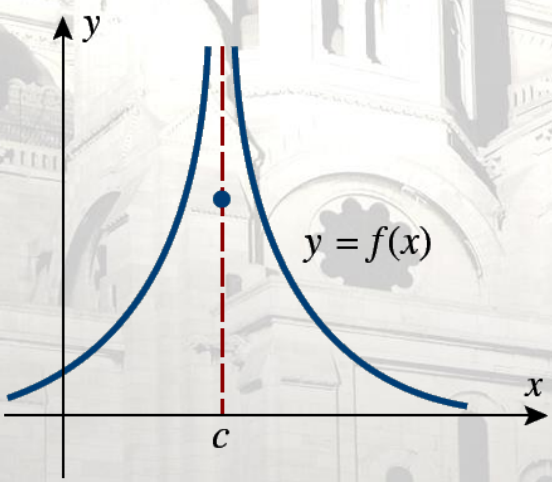
1. $f(c)$ is defined.

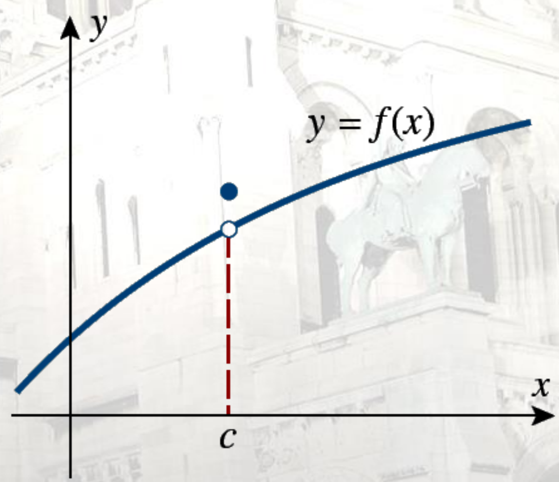
2. $\lim_{x \to c} f(x)$ exists.

3. $\lim_{x \to c} f(x) = f(c)$.

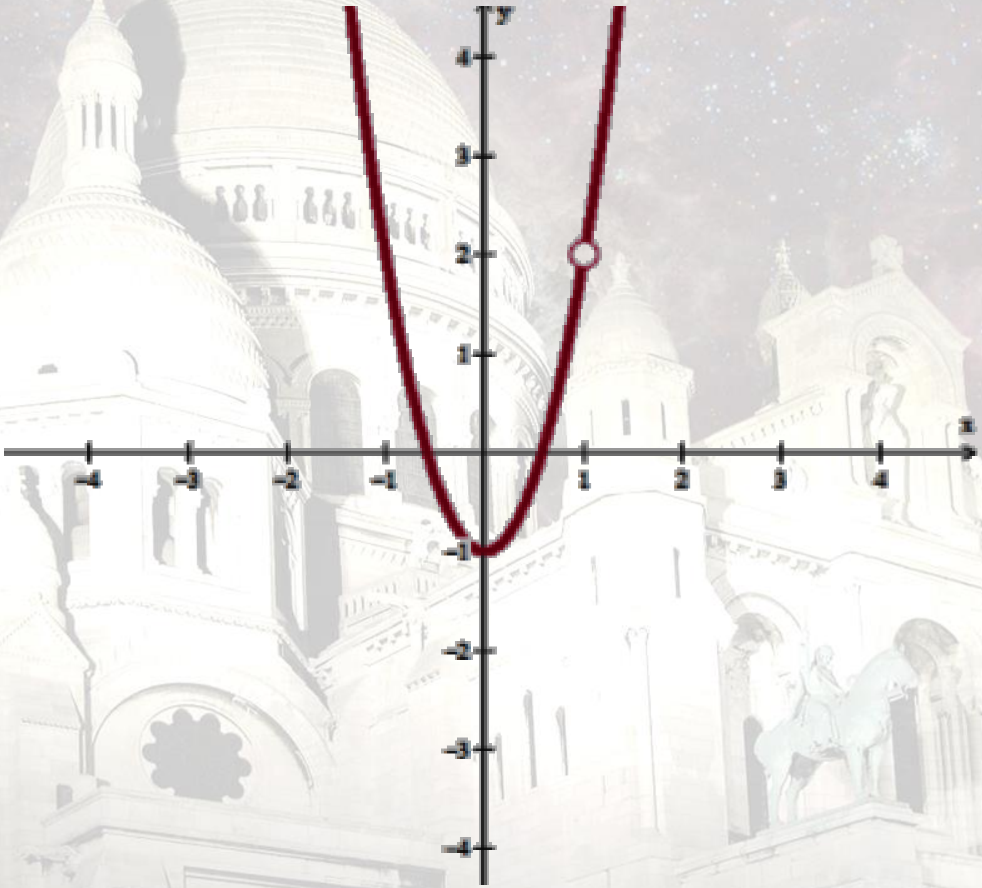








Example 1



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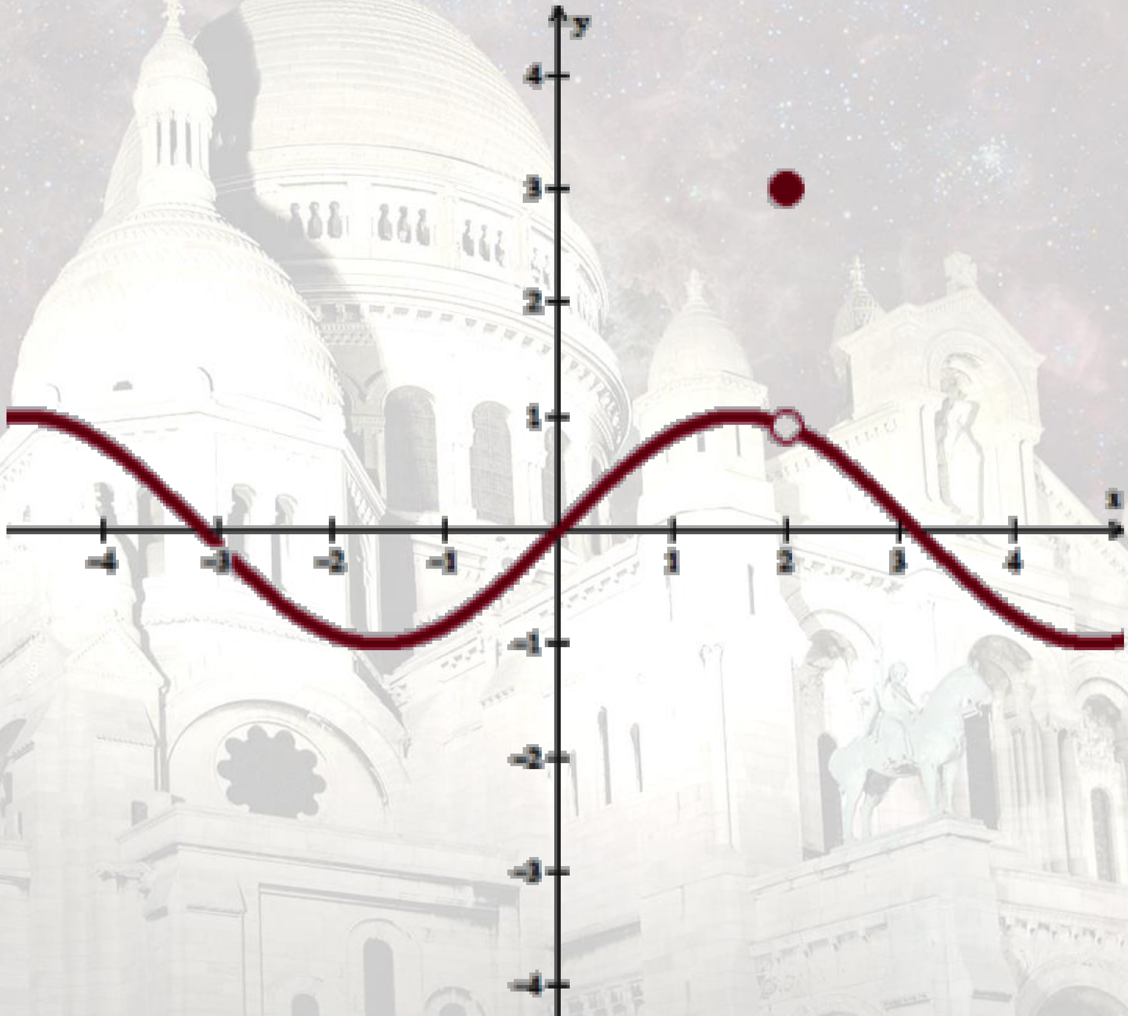

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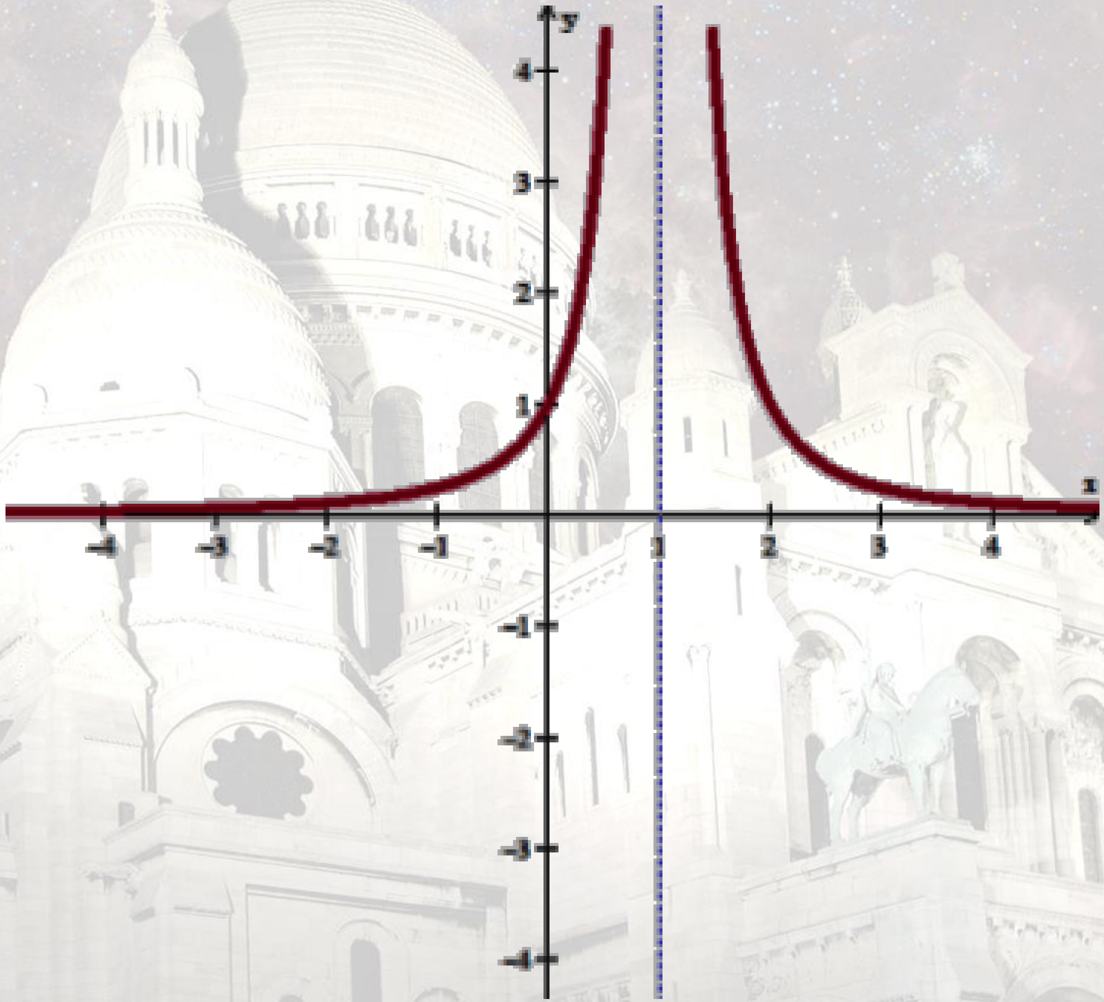

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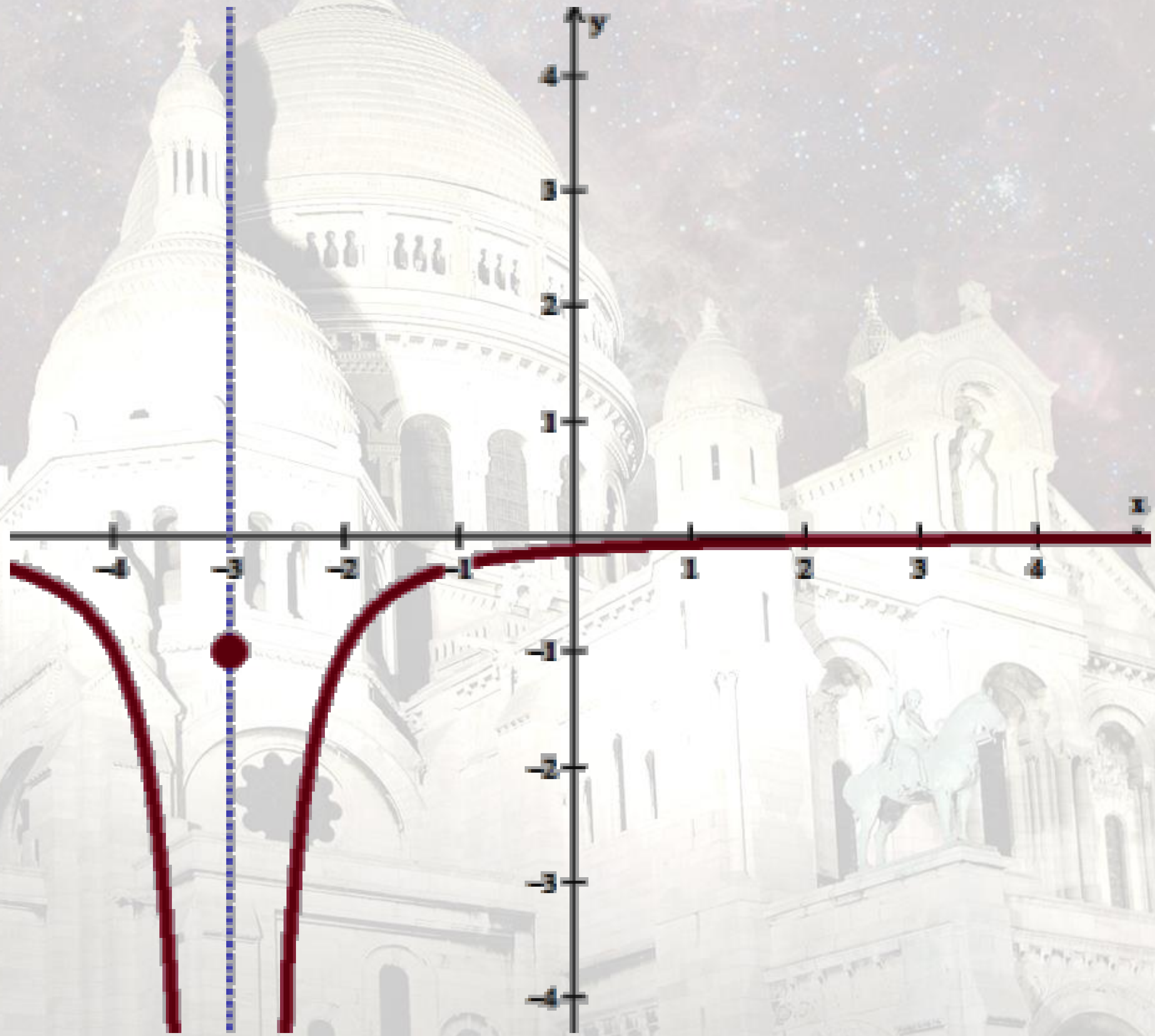

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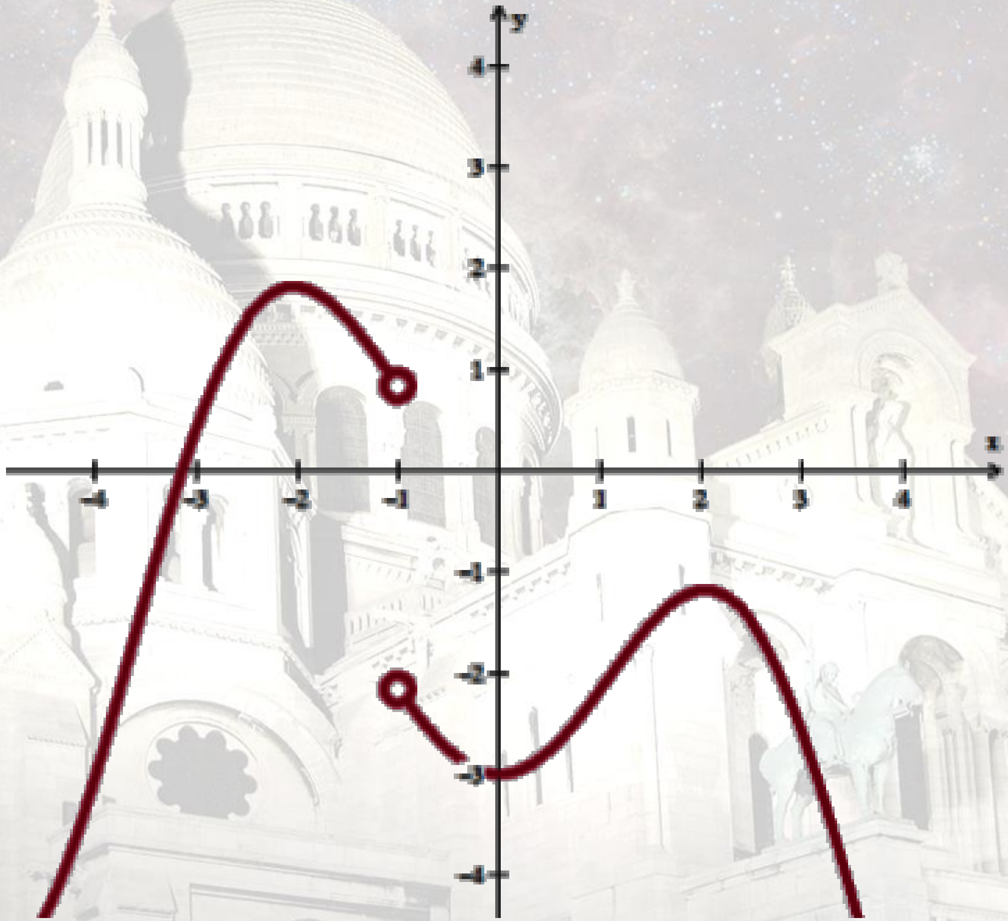
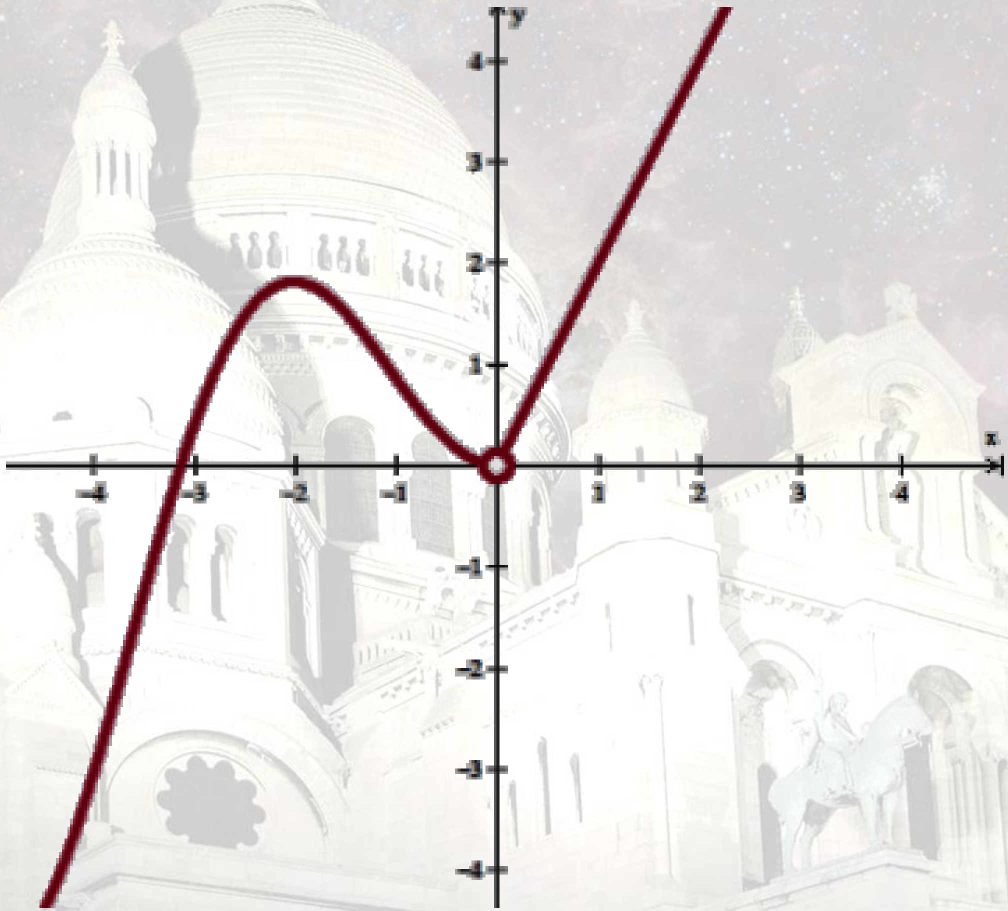


Example 6



Example 7

$$f(x) = \frac{(x-2)(x+4)}{x-2}$$

Example 8

$$f(x) = \frac{3}{x+5}$$

Example 9

$$f(x) = \begin{cases} 2x - 4, & x \geq 1 \\ x^2, & x < 1 \end{cases}$$

Example 10

$$f(x) = \begin{cases} x^2, & x < -4 \\ 2x + 24, & x > -4 \end{cases}$$

Example 11

$$f(x) = \begin{cases} x^2, & x < 10 \\ 150 - \frac{1}{2}x^2, & x = 10 \\ 2x + 80, & x > 10 \end{cases}$$

Joke Time!

- Why did the chicken cross the road?

- Because of the Intermediate Value Theorem

Crossing the Room

- How can I cross the room without getting caught up in the string?

- But let's say I MUST be continuous. Now, how do I get there without getting caught up in the string?

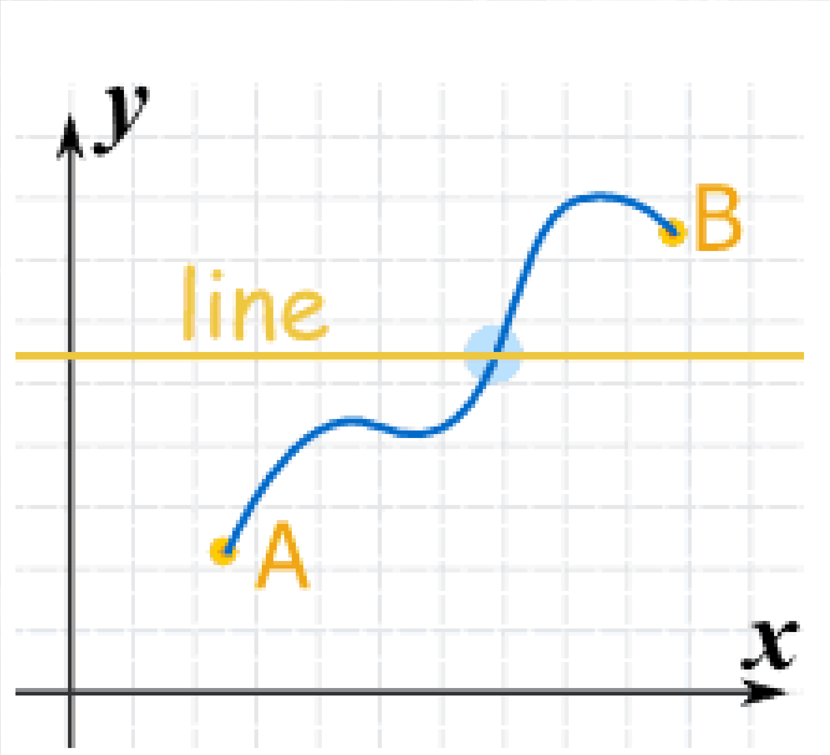
Definition

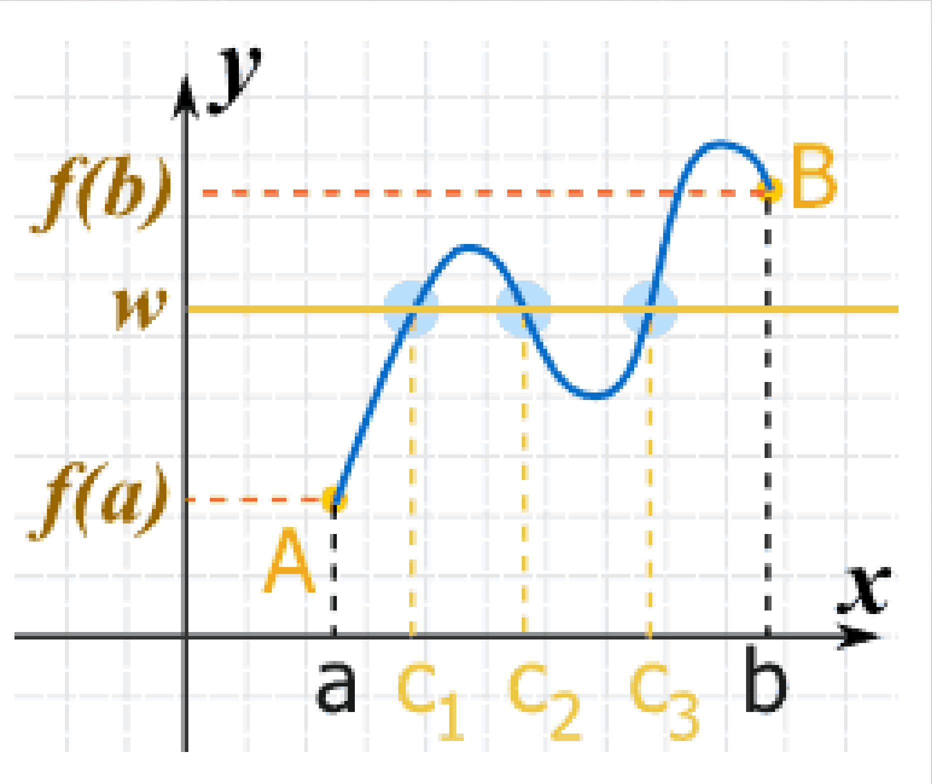
- Intermediate Value Theorem:

- If two points are connected by a continuous curve, every y-value between them has to be crossed at least once.

2.5.7 THEOREM (*Intermediate-Value Theorem*). *If f is continuous on a closed interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, inclusive, then there is at least one number x in the interval $[a, b]$ such that $f(x) = k$.*

Pictures





1. Suppose that a function f is continuous everywhere and that:

$$f(-2) = 3, \quad f(-1) = -1, \quad f(0) = -4, \quad f(1) = 1, \quad \text{and } f(2) = 5.$$

Does the Intermediate Value Theorem guarantee that f has a root on the following intervals?

a. $[-2, -1]$

c. $[-1, 1]$

b. $[-1, 0]$

d. $[0, 2]$

2. Show that the equation $x^3 + x^2 - 2x = 1$ has at least one solution in the interval $[-1, 1]$.

3. A monk begins walking up a mountain road at $12:00$ noon and reaches the top at $12:00$ midnight. He meditates and rests until $12:00$ noon the next day, at which time he begins walking down the same road, reaching the bottom at $12:00$ midnight. Show that there is at least one point on the road that he reaches at the same time of the day on the way up as on the way down.

4. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

a. $x^3 - 3x + 1 = 0, \quad (0,1)$

b. $x^2 = \sqrt{x+1} \quad (1,2)$

c. $\cos x = x, \quad (0,1)$

d. $\ln x = e^{-x}, \quad (1,2)$

5. Prove that the equation has at least one real root.

a. $e^x = 2 - x$

b. $x^5 - x^2 + 2x + 3 = 0$